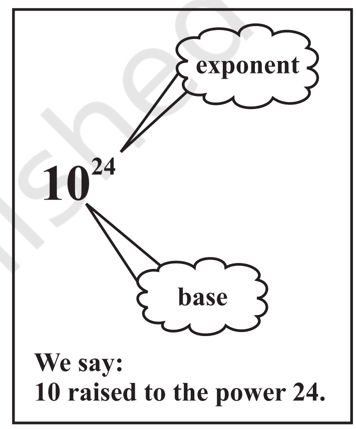
CHAPTER 12

Exponents and Powers

12.1 Introduction



Do you know?

Mass of earth is 5,970,000,000,000, 000, 000, 000, 000 kg. We have already learnt in earlier class how to write such large numbers more conveniently using exponents, as, 5.97×10^{24} kg.

We read 10^{24} as 10 raised to the power 24.

We know $\quad 2^5 = 2 \times 2 \times 2 \times 2 \times 2$

and $\quad 2^m = 2 \times 2 \times 2 \times 2 \times \ldots \times 2 \times 2 \ldots$ (m times)

Let us now find what is 2^{-2} is equal to?

Exponent is a negative integer.

12.2 Powers with Negative Exponents

You know that,

$$10^2 = 10 \times 10 = 100$$

$$10^1 = 10 = \frac{100}{10}$$

$$10^0 = 1 = \frac{10}{10}$$

$$10^{-1} = ?$$

As the exponent decreases by1, the value becomes one-tenth of the previous value.

Continuing the above pattern we get, $10^{-1} = \frac{1}{10}$

Similarly

$$10^{-2} = \frac{1}{10} \div 10 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2}$$

$$10^{-3} = \frac{1}{100} \div 10 = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} = \frac{1}{10^3}$$

What is 10^{-10} equal to?

Now consider the following.

$$3^3 = 3 \times 3 \times 3 = 27$$

$$3^2 = 3 \times 3 = 9 = \frac{27}{3}$$

$$3^1 = 3 = \frac{9}{3}$$

$$3^0 = 1 = \frac{3}{3}$$

The previous number is divided by the base 3.

So looking at the above pattern, we say

$$3^{-1} = 1 \div 3 = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3} \div 3 = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^2} \div 3 = \frac{1}{3^2} \times \frac{1}{3} = \frac{1}{3^3}$$

You can now find the value of 2^{-2} in a similar manner.

We have, $10^{-2} = \frac{1}{10^2}$ or $10^2 = \frac{1}{10^{-2}}$

$10^{-3} = \frac{1}{10^3}$ or $10^3 = \frac{1}{10^{-3}}$

$3^{-2} = \frac{1}{3^2}$ or $3^2 = \frac{1}{3^{-2}}$ etc.

In general, we can say that for any non-zero integer a, $a^{-m} = \frac{1}{a^m}$, where m is a positive integer. a^{-m} is the multiplicative inverse of a^m.

TRY THESE

Find the multiplicative inverse of the following.

(i) 2^{-4} (ii) 10^{-5} (iii) 7^{-2} (iv) 5^{-3} (v) 10^{-100}

We learnt how to write numbers like 1425 in expanded form using exponents as $1 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 5 \times 10^0$.

Let us see how to express 1425.36 in expanded form in a similar way.

We have $1425.36 = 1 \times 1000 + 4 \times 100 + 2 \times 10 + 5 \times 1 + \frac{3}{10} + \frac{6}{100}$

$$= 1 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 5 \times 1 + 3 \times 10^{-1} + 6 \times 10^{-2}$$

$10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

TRY THESE

Expand the following numbers using exponents.

(i) 1025.63 (ii) 1256.249

12.3 Laws of Exponents

We have learnt that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are natural numbers. Does this law also hold if the exponents are negative? Let us explore.

(i) We know that $2^{-3} = \dfrac{1}{2^3}$ and $2^{-2} = \dfrac{1}{2^2}$

$a^{-m} = \dfrac{1}{a^m}$ for any non-zero integer a.

Therefore, $2^{-3} \times 2^{-2} = \dfrac{1}{2^3} \times \dfrac{1}{2^2} = \dfrac{1}{2^3 \times 2^2} = \dfrac{1}{2^{3+2}} = 2^{-5}$

–5 is the sum of two exponents – 3 and – 2

(ii) Take $(-3)^{-4} \times (-3)^{-3}$

$$(-3)^{-4} \times (-3)^{-3} = \frac{1}{(-3)^4} \times \frac{1}{(-3)^3}$$

$$= \frac{1}{(-3)^4 \times (-3)^3} = \frac{1}{(-3)^{4+3}} = (-3)^{-7}$$

$(-4) + (-3) = -7$

(iii) Now consider $5^{-2} \times 5^4$

$$5^{-2} \times 5^4 = \frac{1}{5^2} \times 5^4 = \frac{5^4}{5^2} = 5^{4-2} = 5^{(2)}$$

$(-2) + 4 = 2$

In Class VII, you have learnt that for any non-zero integer a, $\dfrac{a^m}{a^n} = a^{m-n}$, where m and n are natural numbers and $m > n$.

(iv) Now consider $(-5)^{-4} \times (-5)^2$

$$(-5)^{-4} \times (-5)^2 = \frac{1}{(-5)^4} \times (-5)^2 = \frac{(-5)^2}{(-5)^4} = \frac{1}{(-5)^4 \times (-5)^{-2}}$$

$$= \frac{1}{(-5)^{4-2}} = (-5)^{-(2)}$$

$(-4) + 2 = -2$

In general, we can say that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are integers.

TRY THESE

Simplify and write in exponential form.

(i) $(-2)^{-3} \times (-2)^{-4}$ (ii) $p^3 \times p^{-10}$ (iii) $3^2 \times 3^{-5} \times 3^6$

On the same lines you can verify the following laws of exponents, where a and b are non zero integers and m, n are any integers.

(i) $\dfrac{a^m}{a^n} = a^{m-n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m \times b^m = (ab)^m$

(iv) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$ (v) $a^0 = 1$

These laws you have studied in Class VII for positive exponents only.

Let us solve some examples using the above Laws of Exponents.

Example 1: Find the value of

(i) 2^{-3} (ii) $\frac{1}{3^{-2}}$

Solution:

(i) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ (ii) $\frac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

Example 2: Simplify

(i) $(-4)^5 \times (-4)^{-10}$ (ii) $2^5 \div 2^{-6}$

Solution:

(i) $(-4)^5 \times (-4)^{-10} = (-4)^{(5-10)} = (-4)^{-5} = \frac{1}{(-4)^5}$ $\left(a^m \times a^n = a^{m+n},\ a^{-m} = \frac{1}{a^m}\right)$

(ii) $2^5 \div 2^{-6} = 2^{5-(-6)} = 2^{11}$ $(a^m \div a^n = a^{m-n})$

Example 3: Express 4^{-3} as a power with the base 2.

Solution: We have, $4 = 2 \times 2 = 2^2$

Therefore, $(4)^{-3} = (2 \times 2)^{-3} = (2^2)^{-3} = 2^{2 \times (-3)} = 2^{-6}$ $[(a^m)^n = a^{mn}]$

Example 4: Simplify and write the answer in the exponential form.

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$ (ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii) $\frac{1}{8} \times (3)^{-3}$ (iv) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Solution:

(i) $(2^5 \div 2^8)^5 \times 2^{-5} = (2^{5-8})^5 \times 2^{-5} = (2^{-3})^5 \times 2^{-5} = 2^{-15-5} = 2^{-20} = \frac{1}{2^{20}}$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = [(-4) \times 5 \times (-5)]^{-3} = [100]^{-3} = \frac{1}{100^3}$

[using the law $a^m \times b^m = (ab)^m$, $a^{-m} = \frac{1}{a^m}$]

(iii) $\frac{1}{8} \times (3)^{-3} = \frac{1}{2^3} \times (3)^{-3} = 2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \frac{1}{6^3}$

(iv) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4} = (-1)^4 \times 3^4 \times \frac{5^4}{3^4}$

$= (-1)^4 \times 5^4 = 5^4$ $[(-1)^4 = 1]$

Example 5: Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$

Solution: $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

On both the sides powers have the same base different from 1 and – 1, so their exponents must be equal.

Therefore, $m + 6 = 7$

or $m = 7 - 6 = 1$

$a^n = 1$ only if $n = 0$. This will work for any a except $a = 1$ or $a = -1$. For $a = 1$, $1^1 = 1^2 = 1^3 = 1^{-2} = ... = 1$ or $(1)^n = 1$ for infinitely many n. For $a = -1$, $(-1)^0 = (-1)^2 = (-1)^4 = (-1)^{-2} = ... = 1$ or $(-1)^p = 1$ for any even integer p.

Example 6: Find the value of $\left(\frac{2}{3}\right)^{-2}$.

Solution: $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \frac{9}{4}$

$$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$$

In general, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$

Example 7: Simplify (i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

Solution:

(i) $$\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2} = \left\{\frac{1^{-2}}{3^{-2}} - \frac{1^{-3}}{2^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$$

$$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2} = \{9 - 8\} \div 16 = \frac{1}{16}$$

(ii) $$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}} = \frac{5^{-7}}{5^{-5}} \times \frac{8^{-5}}{8^{-7}} = 5^{(-7)-(-5)} \times 8^{(-5)-(-7)}$$

$$= 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25}$$

EXERCISE 12.1

1. Evaluate.

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

3. Find the value of.

(i) $(3^\circ + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\dfrac{-2}{3}\right)^{-2}\right\}^2$

4. Evaluate (i) $\dfrac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

6. Evaluate (i) $\left\{\left(\dfrac{1}{3}\right)^{-1} - \left(\dfrac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\dfrac{5}{8}\right)^{-7} \times \left(\dfrac{8}{5}\right)^{-4}$

7. Simplify.

(i) $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$ (ii) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

12.4 Use of Exponents to Express Small Numbers in Standard Form

Observe the following facts.

1. The distance from the Earth to the Sun is 149,600,000,000 m.
2. The speed of light is 300,000,000 m/sec.
3. Thickness of Class VII Mathematics book is 20 mm.
4. The average diameter of a Red Blood Cell is 0.000007 mm.
5. The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
6. The distance of moon from the Earth is 384, 467, 000 m (approx).
7. The size of a plant cell is 0.00001275 m.
8. Average radius of the Sun is 695000 km.
9. Mass of propellant in a space shuttle solid rocket booster is 503600 kg.
10. Thickness of a piece of paper is 0.0016 cm.
11. Diameter of a wire on a computer chip is 0.000003 m.
12. The height of Mount Everest is 8848 m.

Observe that there are few numbers which we can read like 2 cm, 8848 m, 6,95,000 km. There are some large numbers like 150,000,000,000 m and some very small numbers like 0.000007 m.

Identify very large and very small numbers from the above facts and write them in the adjacent table:

Very large numbers	Very small numbers
150,000,000,000 m	0.000007 m
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We have learnt how to express very large numbers in standard form in the previous class.

For example: $150,000,000,000 = 1.5 \times 10^{11}$

Now, let us try to express 0.000007 m in standard form.

$$0.000007 = \frac{7}{1000000} = \frac{7}{10^6} = 7 \times 10^{-6}$$

$0.000007\text{ m} = 7 \times 10^{-6}\text{ m}$

150000000000. Decimal is moved 11 places to the left.

Similarly, consider the thickness of a piece of paper which is 0.0016 cm.

0.000007 decimal is moved 6 places to the right.

$$0.0016 = \frac{16}{10000} = \frac{1.6 \times 10}{10^4} = 1.6 \times 10 \times 10^{-4}$$

$$= 1.6 \times 10^{-3}$$

Again notice
0.0016 decimal is moved 3 places to the right.

Therefore, we can say thickness of paper is 1.6×10^{-3} cm.

TRY THESE

1. Write the following numbers in standard form.
   (i) 0.000000564 (ii) 0.0000021 (iii) 21600000 (iv) 15240000
2. Write all the facts given in the standard form.

12.4.1 Comparing very large and very small numbers

The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Suppose you want to compare the diameter of the Earth, with the diameter of the Sun.

Diameter of the Sun = 1.4×10^9 m

Diameter of the earth = 1.2756×10^7 m

Therefore $\frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 100}{1.2756}$ which is approximately 100

So, the diameter of the Sun is about 100 times the diameter of the earth.

Let us compare the size of a Red Blood cell which is 0.000007 m to that of a plant cell which is 0.00001275 m.

Size of Red Blood cell = 0.000007 m = 7×10^{-6} m

Size of plant cell = 0.00001275 = 1.275×10^{-5} m

Therefore, $\frac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \frac{7 \times 10^{-6-(-5)}}{1.275} = \frac{7 \times 10^{-1}}{1.275} = \frac{0.7}{1.275} = \frac{0.7}{1.3} = \frac{1}{2}$ (approx.)

So a red blood cell is half of plant cell in size.

Mass of earth is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg. What is the total mass?

Total mass = 5.97×10^{24} kg + 7.35×10^{22} kg.

= $5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22}$

= $597 \times 10^{22} + 7.35 \times 10^{22}$

= $(597 + 7.35) \times 10^{22}$

= 604.35×10^{22} kg.

When we have to add numbers in standard form, we convert them into numbers with the same exponents.

The distance between Sun and Earth is 1.496×10^{11}m and the distance between Earth and Moon is 3.84×10^8m.

During solar eclipse moon comes in between Earth and Sun.

At that time what is the distance between Moon and Sun.

Distance between Sun and Earth = 1.496×10^{11}m
Distance between Earth and Moon = 3.84×10^{8}m
Distance between Sun and Moon = $1.496 \times 10^{11} - 3.84 \times 10^{8}$
$= 1.496 \times 1000 \times 10^{8} - 3.84 \times 10^{8}$
$= (1496 - 3.84) \times 10^{8}$ m $= 1492.16 \times 10^{8}$ m

Example 8: Express the following numbers in standard form.

(i) 0.000035 (ii) 4050000

Solution: (i) $0.000035 = 3.5 \times 10^{-5}$ (ii) $4050000 = 4.05 \times 10^{6}$

Example 9: Express the following numbers in usual form.

(i) 3.52×10^{5} (ii) 7.54×10^{-4} (iii) 3×10^{-5}

Solution:

(i) $3.52 \times 10^{5} = 3.52 \times 100000 = 352000$

(ii) $7.54 \times 10^{-4} = \dfrac{7.54}{10^{4}} = \dfrac{7.54}{10000} = 0.000754$

(iii) $3 \times 10^{-5} = \dfrac{3}{10^{5}} = \dfrac{3}{100000} = 0.00003$

Again we need to convert numbers in standard form into a numbers with the same exponents.

EXERCISE 12.2

1. Express the following numbers in standard form.
   (i) 0.0000000000085 (ii) 0.00000000000942
   (iii) 6020000000000000 (iv) 0.00000000837
   (v) 31860000000
2. Express the following numbers in usual form.
   (i) 3.02×10^{-6} (ii) 4.5×10^{4} (iii) 3×10^{-8}
   (iv) 1.0001×10^{9} (v) 5.8×10^{12} (vi) 3.61492×10^{6}
3. Express the number appearing in the following statements in standard form.
   (i) 1 micron is equal to $\dfrac{1}{1000000}$ m.
   (ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.
   (iii) Size of a bacteria is 0.0000005 m
   (iv) Size of a plant cell is 0.00001275 m
   (v) Thickness of a thick paper is 0.07 mm
4. In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack.

WHAT HAVE WE DISCUSSED?

1. Numbers with negative exponents obey the following laws of exponents.
   (a) $a^{m} \times a^{n} = a^{m+n}$ (b) $a^{m} \div a^{n} = a^{m-n}$ (c) $(a^{m})^{n} = a^{mn}$
   (d) $a^{m} \times b^{m} = (ab)^{m}$ (e) $a^{0} = 1$ (f) $\dfrac{a^{m}}{b^{m}} = \left(\dfrac{a}{b}\right)^{m}$
2. Very small numbers can be expressed in standard form using negative exponents.