

Sample Question Paper

Class XII

Subject: Mathematics

Max. Ms: 80 Max.

Time: 3 Hours

General Instructions:

- I. All questions are compulsory.
- II. This Question Paper has four sections of Section A, B, C, and D.
- III. Section A has all Multiple-Choice Questions. Out of which Q 1 to Q 12, each question carries 1 Mark and has single correct answer.
- IV. Section B has Very Short Answer type questions from Q 13 to Q 27. Each question carries 2 Marks.
- V. Section C has Short Answer type questions from Q 28 to Q 33. Each question carries 3 Marks.
- VI. Section D has Long Answer type questions from Q 34 to Q 38. Each question carries 4 Marks.

| S. No     | Question                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Ms |
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| Section A |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |    |
| 1.        | <p>A telecom company app represents daily data usage using a function <math>f: \mathbb{N} \rightarrow \mathbb{R}^+</math></p> <p>where <math>f(d)</math> gives the megabytes used on day <math>d</math>.<br/>Assume the app records exactly one value of data usage per day.<br/>Based on the relationship between elements in the domain and the codomain, which classification of <math>f</math> is correct?</p> <p>A. It is many-to-one because different days can map to the same usage value.<br/>B. It is one-to-one because each day maps to a different usage value<br/>C. It is onto because every positive real number occurs as a usage value.<br/>D. It is both one-one and onto.</p>                                                                          | 1  |
| Ans       | <p>Correctly identifies that the function is many-to-one, as multiple days may have identical data usage, while there is no given condition ensuring distinct or exhaustive mapping of usage values.</p> <p><b>Justification:</b><br/>The function <math>f: \mathbb{N} \rightarrow \mathbb{R}^+</math> assigns <b>exactly one value</b> of usage to each day <math>\rightarrow</math> ensures it is a <b>function</b>.</p> <ul style="list-style-type: none"><li>Daily data usage can <b>repeat</b> across days <math>\rightarrow</math> allows many-to-one mapping.</li><li>No information guarantees that <b>every</b> positive real number appears as a usage value <math>\rightarrow</math> not onto.</li><li>Therefore, it cannot be both one-one and onto.</li></ul> |    |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |   |
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| 2.  | <p>A student claims:<br/>“For any square matrices <math>A</math> and <math>B</math> of compatible sizes, the rank of the product <math>AB</math> is always equal to the rank of <math>A</math> and the rank of <math>B</math>.”</p> <p>Another student is unsure and wonders if this holds in general.<br/>Which action best supports a mathematical evaluation of this claim for all types of square matrices?</p> <p><b>Options:</b><br/> A. Test the claim using compatible singular, and non-singular square matrices of different sizes.<br/> B. Test the claim using compatible singular, and non-singular square and rectangular matrices of different sizes.<br/> C. Test the claim for squared diagonal matrices of different sizes.<br/> D. Test the claim for square and rectangular diagonal matrices of different sizes.</p>                                                                                                                                                                                               | 1 |
| Ans | <p>Demonstrates recognition that testing both <b>singular and non-singular</b> square matrices provides a comprehensive basis for evaluating the claim across multiple perspectives, aligning with the competency of <b>Exploration</b>.</p> <p><b>Justification</b></p> <ul style="list-style-type: none"> <li>The original claim concerns <b>square matrices</b> only; therefore, including rectangular matrices (as in B or D) does <b>not</b> address the claim correctly.</li> <li>Testing only diagonal matrices (as in C or D) is <b>too narrow</b> and limits the exploration of different structural possibilities.</li> <li>Option <b>A</b> encourages examining the claim across <b>both categories</b> of square matrices—those that are invertible and those that are not—supporting a deeper, more critical evaluation of the assumption.</li> <li>This reflects the essence of <b>Exploration</b>, as it requires the student to question the generality of the claim and investigate a broader set of cases.</li> </ul> | 1 |
| 3.  | <p>A group splits tasks to compute <math>A^{-1} = \frac{1}{\det(A)} \text{adj}(A)</math> for</p> $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 0 & 2 & 1 \end{bmatrix}$ <p>One member computed minors, one cofactors, and one built <math>\text{adj}(A)</math>.</p> <p>Which step should the group take to verify the mathematical correctness of the constructed <math>\text{adj}(A)</math>?</p> <p>A. Combine the minors, cofactors, and adjoint immediately and compute <math>A^{-1}</math>.<br/> B. Confirm <math>A \cdot \text{adj}(A) = \det(A) I</math> with a quick matrix multiplication check. (key)<br/> C. Assign the adjoint construction to the cofactor author for consistency.<br/> D. Recompute minors only and trust the adjoint as built.</p>                                                                                                                                                                                                                                                                          |   |

|    | Option Chosen                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Interpretation                                                                          | Ms   |   |
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|    | B (Correct)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Most appropriate collaborative verification step; mathematically valid and team: based. | 1.00 |   |
|    | A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Some collaborative intent but does not actually verify correctness; partially aligned.  | 0.75 |   |
|    | C                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Shows recognition of team roles but does not check correctness; weak alignment.         | 0.50 |   |
|    | D                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Neither collaborative nor mathematically valid verification.                            | 0.00 |   |
|    | •                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                                                         |      |   |
| 4. | <p>A company wants to choose the production level that maximises profit. A differentiable function <math>P(x)</math> is used to model the profit.</p> <p>The manager believes that <math>x = a</math> is a point of maximum profit based on the model.</p> <p>Which test correctly confirms that <math>x = a</math> is a local maximum of <math>P</math>?</p> <p>A. Verify <math>P'(a) = 0</math> and <math>P''(a) &lt; 0</math>.</p> <p>B. Evaluate <math>P'(x)</math> at randomly chosen points around <math>a</math>.</p> <p>C. Evaluate <math>P(a)</math> and compare it with <math>P(0)</math>.</p> <p>D. Check if the graph of <math>P</math> is symmetric.</p>                                                                                                             |                                                                                         |      | 1 |
|    | <p>Correctly identifies the second derivative test for a local maximum; demonstrates the ability to recall and apply appropriate tests from multiple possible ideas.</p> <p><b>Justification for Correct Answer</b></p> <p>The condition <math>P'(a) = 0</math> identifies a stationary point.</p> <ul style="list-style-type: none"> <li>The condition <math>P''(a) &lt; 0</math> confirms the point is a <b>local maximum</b> by showing concavity.</li> <li>Other options are incorrect: <ul style="list-style-type: none"> <li>Random checking of <math>P'(x)</math> is not systematic.</li> <li>Comparing <math>P(a)</math> with <math>P(0)</math> does not test local maxima.</li> <li>Symmetry of the graph is irrelevant for maxima determination.</li> </ul> </li> </ul> |                                                                                         |      |   |
| 5. | <p>The graph of a continuous function <math>f(x)</math> is such that</p> <ul style="list-style-type: none"> <li><math>f(x) &gt; 0</math> on <math>(0,2)</math>,</li> <li><math>f(x) &lt; 0</math> on <math>(2,4)</math>, and</li> <li><math> f(x) </math> is symmetric about <math>x = 2</math></li> </ul> <p>Which of the following is the most reasonable conclusion about the definite integral</p> $\int_0^4 f(x) dx?$ <p>A. It must be positive because the function is positive on <math>(0,2)</math>, so the area under the curve from 0 to 2 adds a positive value to the integral.</p>                                                                                                                                                                                   |                                                                                         |      | 1 |

|               | <p>B. It must be negative because the function is negative on <math>(2, 4)</math>, so the area under the curve from 2 to 4 subtracts from the integral and 4 is greater than 2.</p> <p>C. It must be zero because the areas above and below the <math>x</math>-axis are equal in magnitude due to the symmetry but opposite in sign, so they cancel each other out.</p> <p>D. It cannot be determined because without knowing the exact values of <math>f(x)</math>, it's impossible to compute the integral.</p>                                                                                                                                                                                        |               |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
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|               | <table> <tr> <th>Option Chosen</th><th>Appropriateness</th><th>Ms</th></tr> <tr> <td>C</td><td>Fully correct — recognizes symmetry of magnitudes about <math>x = 2 \Rightarrow</math> net integral cancels to zero.</td><td>1.00</td></tr> <tr> <td>A</td><td>Partially appropriate — considers only one interval; ignores symmetry information.</td><td>0.75</td></tr> <tr> <td>B</td><td>Minimally appropriate — observes sign on <math>(2, 4)</math> but incorrect reasoning about interval length; ignores symmetry.</td><td>0.50</td></tr> <tr> <td>D</td><td>Incorrect — contradicts the given strong symmetry condition; the conclusion <i>can</i> be determined.</td><td>0.00</td></tr> </table> | Option Chosen | Appropriateness | Ms | C | Fully correct — recognizes symmetry of magnitudes about $x = 2 \Rightarrow$ net integral cancels to zero. | 1.00 | A | Partially appropriate — considers only one interval; ignores symmetry information. | 0.75 | B | Minimally appropriate — observes sign on $(2, 4)$ but incorrect reasoning about interval length; ignores symmetry. | 0.50 | D | Incorrect — contradicts the given strong symmetry condition; the conclusion <i>can</i> be determined. | 0.00 |  |
| Option Chosen | Appropriateness                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Ms            |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
| C             | Fully correct — recognizes symmetry of magnitudes about $x = 2 \Rightarrow$ net integral cancels to zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 1.00          |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
| A             | Partially appropriate — considers only one interval; ignores symmetry information.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 0.75          |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
| B             | Minimally appropriate — observes sign on $(2, 4)$ but incorrect reasoning about interval length; ignores symmetry.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 0.50          |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
| D             | Incorrect — contradicts the given strong symmetry condition; the conclusion <i>can</i> be determined.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 0.00          |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
| 6.            | <p>A water tank is being filled at a rate given by <math>R(t) = 2t</math> litres per minute, where <math>t</math> is the time in minutes and <math>0 \leq t \leq 5</math>.</p> <p>To find the total volume of water collected in the first 5 minutes, which integral should be used?</p> <p>A. <math>\int_0^t 2t \, dt</math></p> <p>B. <math>\int_0^5 2t \, dt</math></p> <p>C. <math>\int_0^t 5t \, dt</math></p> <p>D. <math>\int_0^2 5t \, dt</math></p>                                                                                                                                                                                                                                             | 1             |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
|               | <p><b>Justification</b></p> <ul style="list-style-type: none"> <li>The total volume collected from <math>t = 0</math> to <math>t = 5</math> is given by <math display="block">\int_0^5 R(t) \, dt = \int_0^5 2t \, dt.</math> </li> <li>Option A uses incorrect upper limit (variable <math>t</math> instead of 5).</li> <li>Option C incorrectly replaces the rate <math>2t</math> with <math>5t</math>.</li> <li>Option D uses an incorrect function and incorrect interval.</li> </ul>                                                                                                                                                                                                                |               |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |
| 7.            | <p>A student wants to find the area enclosed by the parabola <math>y = 9 - x^2</math> and the <math>x</math>-axis. After sketching the graph, the student observes that the curve is symmetric about the <math>y</math>-axis and suggests calculating the area from <math>x = 0</math> to <math>x = 3</math> first and doubling the result.</p> <p>Which response is the most appropriate in this situation?</p>                                                                                                                                                                                                                                                                                         | 1             |                 |    |   |                                                                                                           |      |   |                                                                                    |      |   |                                                                                                                    |      |   |                                                                                                       |      |  |

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |   |
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|    | <p>A. “Yes, by symmetry you can compute <math>\int_0^3 (9 - x^2) dx</math> and multiply the result by 2.”</p> <p>B. “Yes, by symmetry you can compute <math>\int_0^3 (9 - x^2) dx</math> and multiply the result by 2, but because the parabola opens upward you then need to multiply the result by <math>-1</math>.”</p> <p>C. “No, compute the left half only because symmetry does not apply to parabolas and the function is negative on the right side.”</p> <p>D. “No, always compute definite integrals from left to right; symmetry cannot be used for area.”</p>                                                                       |   |
|    | <p><b>Option Chosen</b>    <b>Appropriateness</b></p> <p><b>A</b>            Fully appropriate — correctly integrates ideas of symmetry, area under curves, and definite integrals.    1</p> <p><b>B</b>            Partially appropriate — recognizes symmetry but adds a mathematically incorrect modification; shows partial integration of concepts.    0.75</p> <p><b>C</b>            Minimally appropriate — rejects symmetry incorrectly; misunderstands sign of function.    0.5</p> <p><b>D</b>            Completely inappropriate — contradicts standard calculus principles.    0</p>                                               |   |
| 8. | <p>A student tries to solve the differential equation</p> $(x + y) dx - x dy = 0$ <p>but finds it cannot be separated directly in its current form.<br/>Which alternate method converts the equation into a separable differential equation?</p> <p>A. Convert to a linear differential equation in <math>y</math> which results in a separable differential equation.</p> <p>B. Apply integrating factor to the original equation.</p> <p>C. Set <math>y = vx</math> and rewrite the equation in terms of <math>v</math> and <math>x</math>.</p> <p>D. Differentiate both sides again to obtain a separable equation of higher order.</p>       | 1 |
|    | <p>Correct option C: substitution <math>y = vx</math> reduces the equation to a homogeneous form, making it separable.</p> <p><b>Justification for Evaluators</b></p> <ul style="list-style-type: none"> <li>The differential equation <math>(x + y) dx - x dy = 0</math> can be rewritten as</li> </ul> $\frac{dy}{dx} = 1 + \frac{y}{x}.$ <p>This is a <b>homogeneous differential equation</b>.</p> <ul style="list-style-type: none"> <li>Using substitution <math>y = vx \Rightarrow dy/dx = v + x dv/dx</math> converts it into a separable form.</li> <li>Therefore, <b>Option C</b> is the correct <i>alternate strategy</i>.</li> </ul> |   |
| 9. | <p>A learner insists the only valid way to find <math>\vec{a} + \vec{b}</math> is the “triangle law of addition.”</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 1 |

|               | <p>Which statement correctly broadens valid methods for finding the same resultant vector?</p> <p>A. The parallelogram law also provides a valid method for finding resultant vector.</p> <p>B. The triangle law is the only valid method for vector addition.</p> <p>C. Resultants cannot be computed without coordinate geometry tools</p> <p>D. Component- wise addition is invalid unless vectors lie on coordinate axes.</p>                                                                                                                                                                                                                                                                                                                                                                                      |               |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
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|               | Option A: Correctly identifies an alternate geometric perspective the parallelogram law, which yields the same resultant vector. Reflects ability to broaden methods and challenge the assumption that only one approach is valid.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |               |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
| 10.           | <p>A robotic arm movement direction <math>\vec{u}</math> must trigger a stop when it becomes nearly perpendicular to a reference direction <math>\vec{v}</math> for safety.</p> <p>Which rule best implements this perpendicularity threshold?</p> <p>A. Monitor <math> \vec{u} \cdot \vec{v} </math> and stop when <math> \vec{u} \cdot \vec{v}  \leq \varepsilon</math> for a small tolerance <math>\varepsilon &gt; 0</math>.</p> <p>B. Stop whenever <math>\ \vec{u}\  = \ \vec{v}\ </math>.</p> <p>C. Use the cross product only; perpendicularity is determined solely by <math>\ \vec{u} \times \vec{v}\ </math>.</p> <p>D. Stop the arm only when <math>\vec{u} = \vec{v}</math>.</p>                                                                                                                          | 1             |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
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| Option Chosen | Appropriateness                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | Ms            |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
| A             | Fully appropriate and innovative: uses dot product magnitude with tolerance to detect “near: perpendicular” orientation, a realistic engineering safety rule.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 1.00          |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
| C             | Partially appropriate: uses a valid perpendicularity indicator, but ignores the <i>near</i> condition and tolerance concept—less innovative than A.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 0.75          |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
| B             | Minimally appropriate: relates vector lengths, but has no connection to direction or perpendicularity.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 0.50          |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
| D             | Completely inappropriate: equating vectors means same direction, not perpendicular.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 0.00          |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
| 11.           | <p>A farmer plants Crop P and Crop Q. Each unit of P requires 1.5 units of land; each unit of Q needs 2 units of land. The total available land is 80 units.</p> <p>Which constraint correctly represents this land- capacity condition?</p> <p>A. <math>P + Q \leq 80</math></p> <p>B. <math>1.5P + 2Q \leq 80</math></p> <p>C. <math>1.5P + Q = 80</math></p> <p>D. <math>P + 2Q &lt; 80</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                  | 1             |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |
|               | <p><b>Option B is correct</b></p> <ul style="list-style-type: none"> <li>Correctly applies the idea that total land used = <math>(1.5 \times \text{units of P}) + (2 \times \text{units of Q})</math></li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                 |    |   |                                                                                                                                                               |      |   |                                                                                                                                                     |      |   |                                                                                                        |      |   |                                                                                     |      |  |

|                  | <ul style="list-style-type: none"> <li>Recognizes real-world constraints must use <math>\leq</math> (not = or <math>&lt;</math>).</li> <li>Demonstrates the ability to combine contextual data with mathematical modelling key to generating innovative solutions in LPP.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |               |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
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| 12.              | <p>Two 3:D lines have direction vectors <math>\vec{d}_1 = (1, -2, 1)</math> and <math>\vec{d}_2 = (2, 1, 3)</math>, respectively. An engineer assumes the shortest segment from one line to the other must lie along <math>\vec{d}_1</math>.</p> <p>Before proceeding, which check is most appropriate to identify the correct direction for the shorter distance between the two lines?</p> <p>A. Compute a few sample distances along <math>\vec{d}_1</math> without checking whether a perpendicular line exists.</p> <p>B. Compute <math>\vec{d}_1 \times \vec{d}_2</math> to verify whether a perpendicular direction is possible.</p> <p>C. Continue using <math>\vec{d}_1</math> only, since one direction is sufficient to estimate the minimum distance.</p> <p>D. Assume the lines are parallel lines without verifying the actual directional relationship.</p>                                                                                                                                                                                                                                                                                                                        | 1             |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
|                  | <table> <tr> <th>Option Chosen</th><th>Appropriateness</th><th>Ms</th></tr> <tr> <td></td><td>The shortest distance between skew (or nonparallel) lines lies along a line <b>perpendicular to both</b> direction vectors.</td><td></td></tr> <tr> <td></td><td>Checking perpendicularity using</td><td></td></tr> <tr> <td><b>B</b></td><td><math>\vec{d}_1 \times \vec{d}_2</math></td><td><b>1.00</b></td></tr> <tr> <td></td><td>is the correct geometric method.</td><td></td></tr> <tr> <td></td><td>Attempts distance evaluation but <b>fails</b> to question perpendicularity, which is essential.</td><td></td></tr> <tr> <td><b>A</b></td><td>Shows some exploratory intent but not mathematically sound.</td><td><b>0.75</b></td></tr> <tr> <td></td><td>Reflects <b>no exploration</b> continues with an assumption without verification.</td><td></td></tr> <tr> <td><b>C</b></td><td>Minimal alignment with the competency.</td><td><b>0.50</b></td></tr> <tr> <td></td><td>Incorrectly introduces a <b>false assumption</b> (parallelism).</td><td></td></tr> <tr> <td><b>D</b></td><td>Does not examine or challenge the given information.</td><td><b>0.00</b></td></tr> </table> | Option Chosen | Appropriateness | Ms |  | The shortest distance between skew (or nonparallel) lines lies along a line <b>perpendicular to both</b> direction vectors. |  |  | Checking perpendicularity using |  | <b>B</b> | $\vec{d}_1 \times \vec{d}_2$ | <b>1.00</b> |  | is the correct geometric method. |  |  | Attempts distance evaluation but <b>fails</b> to question perpendicularity, which is essential. |  | <b>A</b> | Shows some exploratory intent but not mathematically sound. | <b>0.75</b> |  | Reflects <b>no exploration</b> continues with an assumption without verification. |  | <b>C</b> | Minimal alignment with the competency. | <b>0.50</b> |  | Incorrectly introduces a <b>false assumption</b> (parallelism). |  | <b>D</b> | Does not examine or challenge the given information. | <b>0.00</b> |  |
| Option Chosen    | Appropriateness                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Ms            |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
|                  | The shortest distance between skew (or nonparallel) lines lies along a line <b>perpendicular to both</b> direction vectors.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |               |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
|                  | Checking perpendicularity using                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |               |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
| <b>B</b>         | $\vec{d}_1 \times \vec{d}_2$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | <b>1.00</b>   |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
|                  | is the correct geometric method.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |               |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
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| <b>A</b>         | Shows some exploratory intent but not mathematically sound.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | <b>0.75</b>   |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
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|                  | Incorrectly introduces a <b>false assumption</b> (parallelism).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |               |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
| <b>D</b>         | Does not examine or challenge the given information.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | <b>0.00</b>   |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
| <b>Section B</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |               |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |
| 13.              | <p>Let</p> $f: A \rightarrow B, g: B \rightarrow C,$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 2             |                 |    |  |                                                                                                                             |  |  |                                 |  |          |                              |             |  |                                  |  |  |                                                                                                 |  |          |                                                             |             |  |                                                                                   |  |          |                                        |             |  |                                                                 |  |          |                                                      |             |  |

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|     | <p>and suppose <math>g \circ f: A \rightarrow C</math> is bijective.<br/> Student X concludes that both <math>f</math> and <math>g</math> must be bijective.<br/> Student Y claims that only the composition needs to be bijective.<br/> Provide a conclusion that addresses these two views using properties of functions.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |   |
|     | <p><b>(Correct functional reasoning (both points required): (1M)</b></p> <ul style="list-style-type: none"> <li>Since <math>g \circ f</math> is <b>injective</b>, <math>f</math> must be <b>injective</b>.</li> <li>Since <math>g \circ f</math> is <b>surjective</b>, <math>g</math> must be <b>surjective</b>.</li> </ul> <p><b>(2) Clear resolution of the conflict between Student X and Student Y: (1M)</b></p> <ul style="list-style-type: none"> <li>State that <b>Student X is only partially correct</b> because neither function needs to be bijective individually.</li> <li>State that <b>Student Y's view is incomplete</b>, because even though only the composition is given bijective, it forces specific properties on <math>f</math> and <math>g</math>.</li> <li>Final conclusion: <ul style="list-style-type: none"> <li><math>f</math> must be <b>injective</b>,</li> <li><math>g</math> must be <b>surjective</b>,</li> <li>but <b>neither is required to be bijective</b>.</li> </ul> </li> </ul> |   |
| 14. | <p>A student claims: “A symmetric matrix mirrors itself and a matrix that mirrors itself is symmetric.”<br/> Explain the <i>meaning</i> of this statement using the relation between <math>a_{ij}</math> and <math>a_{ji}</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 2 |
|     | <p><b>Point 1: Understanding the mirroring idea (1 M)</b></p> <ul style="list-style-type: none"> <li>States that for a matrix to “mirror itself,” <b>each entry equals its transpose counterpart</b>, i.e.,<br/> <math>a_{ij} = a_{ji}</math>.</li> <li>Shows comprehension that the matrix reflects across its <b>main diagonal</b>.</li> </ul> <p><b>Point 2 : Defining the construct of a symmetric matrix (1 M)</b></p> <ul style="list-style-type: none"> <li>States that a matrix satisfying <math>a_{ij} = a_{ji}</math> <b>for all i, j</b> is called <b>symmetric</b>, i.e.,<br/> <math>A = A^T</math>.</li> <li>Demonstrates understanding that this <i>property itself defines the conceptual structure</i> of symmetric matrices.</li> </ul>                                                                                                                                                                                                                                                                 |   |
| 15. | <p>Use an elementary row or column operation to simplify the matrix</p> $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 1 & 4 & 7 \end{vmatrix}$ <p>and then compute the determinant of the matrix.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 2 |

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|     | <p><b>Point 1 : Correct elementary operation applied (1 M)</b><br/>Any ONE valid operation must be shown. Examples include:</p> <ul style="list-style-type: none"> <li>• <math>R_2 \rightarrow R_2 - 2R_1</math></li> <li>• <math>R_3 \rightarrow R_3 - R_1</math></li> <li>• Or a similar correct column operation.</li> </ul> <p><b>Example correct operation:</b></p> $R_2 \rightarrow R_2 - 2R_1 \Rightarrow \begin{vmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \\ 1 & 4 & 7 \end{vmatrix}$ <p><b>Point 2 : Correct simplified conclusion (1 M)</b><br/>After a valid operation, the student should recognize:</p> <ul style="list-style-type: none"> <li>• The determinant has a <b>row of zeros</b>,</li> <li>• Therefore,</li> </ul> $\det = 0.$ <p>(Any equivalent simplified determinant showing the result is acceptable.)</p>                                                                                         |   |
| 16. | <p>Let a function <math>f(x)</math> represent the velocity of a particle moving along a straight line and suppose <math>f(x)</math> shows periods where its rate of increase sharply accelerates. Describe how examining the sign and magnitude of <math>f'(x)</math> in relation to <math>f(x)</math> itself can help predict intervals where the particle may be approaching unstable motion.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2 |
|     | <p><b>Point 1: Interpreting sign and magnitude of <math>f'(x)</math> in an innovative way (1 M)</b></p> <ul style="list-style-type: none"> <li>• A <b>positive and rapidly increasing</b> <math>f'(x)</math> suggests acceleration growing faster than usual.</li> <li>• Large magnitude of <math>f'(x)</math> compared with <math>f(x)</math> signals <b>disproportionate growth</b> in velocity.</li> </ul> <p><b>Point 2: Creative prediction of unstable motion (1 M)</b></p> <ul style="list-style-type: none"> <li>• When <math>f'(x)</math> becomes <b>large relative to</b> <math>f(x)</math>, the motion may become unstable (velocity changing too rapidly).</li> <li>• Such intervals indicate <b>loss of control</b>, “runaway” behaviour, or approaching <b>instability zones</b>.</li> <li>• The student extends mathematical understanding to <b>predict nonlinear physical behaviour</b>.</li> </ul> |   |
| 17. | <p>A manufacturer models the efficiency <math>E(x)</math> (in percent) of a machine as a function of the operating temperature <math>x</math> (in degrees Celsius):</p> $E(x) = x^3 - 3x + 1,$ <p>An engineer claims that the machine’s efficiency has no maximum point.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | 2 |

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|     | Using calculus, explain why this claim should be reconsidered, and determine the temperature(s) at which machine has a local maximum or minimum efficiency.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |   |
|     | <p><b>Point 1 : Demonstrates openness to revising the initial belief using derivative reasoning (1 M)</b></p> <ul style="list-style-type: none"> <li>Notes that the <b>initial visual conclusion</b> (“no maximum”) may be incorrect.</li> <li>Uses derivative evidence to <b>challenge and revise</b> this assumption.</li> <li>Computes the derivative: <math display="block">E'(x) = 3x^2 - 3 = 3(x^2 - 1).</math> </li> <li>Identifies critical points correctly: <math display="block">E'(x) = 0 \Rightarrow x = \pm 1.</math> </li> </ul> <p><b>Point 2 : Determines nature of critical points using the second derivative or sign analysis (1 M)</b></p> <ul style="list-style-type: none"> <li>Computes second derivative: <math display="block">E''(x) = 6x.</math> </li> <li>Evaluates at critical points: <ul style="list-style-type: none"> <li><math>E''(-1) = -6 &lt; 0 \rightarrow</math> <b>local maximum</b></li> <li><math>E''(1) = 6 &gt; 0 \rightarrow</math> <b>local minimum</b></li> </ul> </li> <li>Concludes that the earlier assumption was incorrect because calculus reveals a <b>local maxima and minima</b> even if the graph initially misled them.</li> </ul> |   |
| 18. | <p>The graph of a function <math>f(x)</math> near <math>x = 3</math> shows the following features:</p> <ul style="list-style-type: none"> <li>As <math>x</math> approaches 3 from the left, <math>f(x)</math> approaches 2.</li> <li>As <math>x</math> approaches 3 from the right, <math>f(x)</math> also approaches 2.</li> <li><math>f(3) = 4</math>.</li> </ul> <p>Based on what the graph conveys, decide whether the function is continuous at <math>x = 3</math>. Briefly justify your answer.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 2 |
|     | <p><b>Point 1 : Uses multiple perspectives to identify the limit correctly (1 M)</b></p> <ul style="list-style-type: none"> <li>Concluding from both one-sided behaviours that <math display="block">\lim_{x \rightarrow 3} f(x) = 2.</math> </li> <li>Demonstrating the ability to shift between the <i>graphical approach</i> and the <i>formal limit definition</i>.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |   |

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|     | <p><b>Point 2 : Compares limit and function value to decide continuity (1 M)</b></p> <ul style="list-style-type: none"> <li>Recognises that the value shown on the graph is</li> </ul> $f(3) = 4.$ <ul style="list-style-type: none"> <li>Concludes that, since</li> </ul> $\lim_{x \rightarrow 3} f(x) = 2 \neq f(3),$ <p>the function is <b>not continuous</b> at <math>x = 3</math>.</p> <ul style="list-style-type: none"> <li>Shows conceptual flexibility by shifting from graphical interpretation to symbolic reasoning.</li> </ul>                                                                                                                                                                                                                                                                                                                                        |   |
| 19. | Evaluate whether Rolle's theorem is applicable to the function $f(x) =  x $ , for $x \in (-2, 2)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 2 |
|     | <p><b>Point 1 : Checks all three conditions of Rolle's Theorem (1 M)</b></p> <ul style="list-style-type: none"> <li><b>Continuity:</b> <math> x </math> is <b>discontinuous at every integer</b>, hence <b>not continuous</b> on <math>[5, 9]</math>.<br/>Discontinuities occur at <math>x = 5, 6, 7, 8, 9</math>.</li> <li>Therefore, the first condition <b>fails</b>.</li> </ul> <p>(Students may also mention nondifferentiable, which further confirms failure.)</p> <p><b>Point 2 : Concludes Rolle's Theorem is NOT applicable (1 M)</b></p> <ul style="list-style-type: none"> <li>Since the function is <b>not continuous</b> on <math>[5, 9]</math>, Rolle's Theorem <b>does not apply</b>.</li> <li>(Optionally) Even though <math>f(5) = 5</math> and <math>f(9) = 9</math>, the theorem fails because the most essential condition continuity is violated.</li> </ul> |   |
| 20. | <p>Consider the integral</p> $\int \frac{5x - 3}{x^2 - 3x + 2} dx.$ <p>Explain why the method of integration by partial fractions is appropriate for evaluating this integral. In your answer, refer to the structure of the integrand and the conditions under which partial fractions can be used.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 2 |
|     | <p><b>Point 1: Identifies key structural features of the integrand (1 M)</b></p> <ul style="list-style-type: none"> <li>The integrand is a <b>rational function</b>, i.e., a polynomial divided by another polynomial.</li> <li>The denominator</li> </ul> $x^2 - 3x + 2 = (x - 1)(x - 2)$ <p><b>factors into linear factors.</b></p> <ul style="list-style-type: none"> <li>The degree of the numerator (1) is <b>less than</b> the degree of the denominator (2).</li> </ul> <p><b>Point 2 : Explains why these features justify using partial fractions (1 M)</b></p>                                                                                                                                                                                                                                                                                                           |   |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |
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|     | <ul style="list-style-type: none"> <li>Because the rational function has a factorable denominator composed of <b>simple linear factors</b>, it can be decomposed into <math display="block">\frac{A}{x-1} + \frac{B}{x-2}.</math> </li> <li>This decomposition converts the integral into a <b>sum of simpler standard integrals</b> (logarithmic forms).</li> <li>Partial fractions is therefore <b>the most appropriate constructive method</b>.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |   |
| 21. | <p>Two classmates disagree on the method to find the indefinite integral:</p> $\int (x^2 + 1)e^x dx.$ <p>One prefers to use substitution, while the other suggests to use repeated integration by parts.</p> <p>Describe which of these two integration techniques is more suitable for this integral and justify your choice.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 2 |
|     | <p><b>Point 1 : Identifies the type/source of conflict and evaluates both methods (1 M)</b></p> <ul style="list-style-type: none"> <li>Substitution is <b>not helpful</b>, because substituting for <math>x^2 + 1</math> or <math>e^x</math> does not simplify the integrand.</li> <li>Integration by parts is appropriate because the integrand is a product of a polynomial and an exponential function.</li> <li>Repeated application of integration by parts systematically reduces the power of <math>x</math>.</li> <li>Clearly shows understanding of <i>why</i> the disagreement arises (different method preferences).</li> </ul> <p><b>Point 2 : Provides a constructive conclusion resolving the disagreement (1 M)</b></p> <ul style="list-style-type: none"> <li><b>Integration by parts is the suitable method</b>, because: <ul style="list-style-type: none"> <li>The integrand fits the standard form: <i>polynomial</i> <math>\times</math> <i>exponential function</i>.</li> <li>Each application of parts lowers the power of <math>x</math>, making the integral solvable.</li> </ul> </li> <li>Acknowledges that the preference for substitution is understandable but <b>not effective</b> here.</li> <li>Provides a <b>balanced resolution</b> that integrates both viewpoints before deciding.</li> </ul> |   |
| 22. | <p>A student claims that definite integrals always produce positive values because integration “finds area.”</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 2 |

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|     | <p>After examining the function <math>f(x) = x - 3</math> on the interval <math>[0,5]</math>, another student disagrees.</p> <p>Explain how considering this function can help refine the original claim about definite integrals and areas.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |   |
|     | <p><b>Point 1 : Acknowledges the new perspective and recognises why the initial assumption is incomplete (1 M)</b></p> <ul style="list-style-type: none"> <li>Notes that the learner's assumption ("integrals always give positive area") is <b>not always correct</b>.</li> <li>Recognises that the second student introduces <b>new evidence</b> worth considering — the graph of <math>f(x) = x - 3</math>.</li> <li>States that the function is <b>below the x-axis</b> on part of <math>[0,5]</math>, causing negative contributions.</li> </ul> <p><b>Point 2 : Uses the example to refine the assumption with correct reasoning (1 M)</b></p> <ul style="list-style-type: none"> <li>On <math>[0,3]</math>, <math>f(x) = x - 3 &lt; 0</math>, so the integral accumulates <b>negative signed area</b>.</li> <li>On <math>[3,5]</math>, <math>f(x) \geq 0</math>, giving positive area.</li> <li>Therefore, the <b>definite integral represents signed area</b>, not always positive area.</li> <li>This evidence <b>modifies</b> the original belief in a constructive way.</li> </ul> |   |
| 23. | <p>Two students examine the differential equation</p> $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} = 0.$ <p>One points out the highest derivative present, and another observes whether the equation is polynomial in derivatives.</p> <p>Explain how these observations help determine order and degree of this differential equation.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 2 |
|     | <p><b>Point 1 : Uses the first student's observation to determine the order (1 M)</b></p> <ul style="list-style-type: none"> <li>The <b>highest order derivative</b> present is</li> </ul> $\frac{d^3y}{dx^3},$ <p>so the <b>order of the differential equation is 3</b>.</p> <ul style="list-style-type: none"> <li>Acknowledges that identifying the highest derivative is a <b>key collaborative contribution</b>.</li> </ul> <p><b>Point 2 — Uses the second student's observation to determine the degree (1 M)</b></p> <ul style="list-style-type: none"> <li>The differential equation is <b>polynomial in derivatives</b> because</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                          |   |

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|     | $\left(\frac{d^3y}{dx^3}\right)^2$ <p>is a polynomial term and does not involve radicals, fractional powers, or nonpolynomial forms.</p> <ul style="list-style-type: none"> <li>Since the highest order derivative is raised to <b>power 2</b>, the <b>degree is 2</b>.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |   |
| 24. | <p>A physics student models the fall of a small object in air by stating: “The acceleration of the object is proportional to its velocity.”</p> <p>Write the first order differential equation that represents this situation.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 2 |
|     | <p><b>Point 1 : Identifies acceleration as the derivative of velocity (1 M)</b></p> $\frac{dv}{dt} = \text{acceleration.}$ <p><b>Point 2 :Writes the correct proportional relationship as a differential equation (1 M)</b></p> $\frac{dv}{dt} = kv$ <p>or</p> $\frac{dv}{dt} \propto v$ <p>or</p> $\frac{dv}{dt} = -kv$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |   |
| 25. | <p>Two classmates are checking whether events <math>A</math> and <math>B</math> are independent. One uses <math>P(A \cap B) = P(A)P(B)</math>, while the other wants to verify if <math>P(B   A) = P(B)</math>.</p> <p>Explain how both methods can be used to confirm independence and describe the relationship between them.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 2 |
|     | <p><b>Point 1 : Shows how both students’ contributions are valid &amp; complementary (1 M)</b></p> <ul style="list-style-type: none"> <li><b>Both conditions express independence</b>, but from different viewpoints — one through <b>joint probability</b>, the other through <b>conditional probability</b>.</li> <li>Students can compute <math>P(A \cap B)</math>, <math>P(A)</math>, and <math>P(B)</math>, then verify if <math display="block">P(A \cap B) = P(A)P(B).</math> </li> </ul> <p><b>Point 2 : Explains how they can jointly use the conditional form to confirm independence (1 M)</b></p> <ul style="list-style-type: none"> <li>After computing the probabilities, they can also check <math display="block">P(B   A) = \frac{P(A \cap B)}{P(A)}</math> </li> </ul> <p>and confirm whether it equals <math>P(B)</math>.</p> |   |

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|     | <ul style="list-style-type: none"> <li>Concluding that <b>both checks lead to the same result</b>, and using both strengthens clarity and agreement.</li> <li>Emphasises collaborative synthesis: <b>two different strategies leading to the same decision</b> about independence.</li> </ul>                                                                                                                                                                                                  |   |
| 26. | <p>Given:</p> $P(A) = 0.3, P(B) = 0.5, P(A \cap B) = 0.12.$ <p>A student first checks independence by verifying</p> $P(A \cap B) = P(A)P(B).$ <p>If they prefer using conditional probability instead, what value should they compute, and how does this help determine independence?</p>                                                                                                                                                                                                      | 2 |
|     | <p><b>Point 1 : Identifies correct conditional probability to compute (1 M)</b></p> $P(B   A) = \frac{P(A \cap B)}{P(A)}.$ <p>Substitute the values:</p> $P(B   A) = \frac{0.12}{0.3} = 0.4.$ <p><b>Point 2 : Compares with <math>P(B)</math> to conclude independence (1 M)</b></p> $P(B   A) = 0.4 \neq 0.5 = P(B),$ <p>so the events <b>are not independent</b>.</p>                                                                                                                        |   |
| 27. | <p>Two students compute the direction cosines of a line.</p> <p>Student A gets: <math>(l, m, n) = (\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}})</math>.</p> <p>Student B gets: <math>(l, m, n) = (-\frac{3}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}})</math>.</p> <p>Both claim their answers are correct.</p> <p>Verify by calculation whether both sets can represent the same line direction, and justify whether both answers may be acceptable</p>        | 2 |
|     | <p><b>Point 1 : Verifies correctness through calculation (1 M)</b></p> <ul style="list-style-type: none"> <li>Both sets satisfy the direction cosine condition:</li> </ul> $l^2 + m^2 + n^2 = \frac{9 + 4 + 1}{14} = 1, \text{ and similarly for the negatives.}$ <ul style="list-style-type: none"> <li>Therefore, <b>both triplets are valid direction cosines</b>.</li> </ul> <p><b>Point 2 : Shows open-minded reasoning: opposite direction cosines represent the same line (1 M)</b></p> |   |

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|                  | <ul style="list-style-type: none"> <li>The two sets differ only by a <b>negative sign</b>:<br/> <math>(-l, -m, -n)</math> represents the same line but opposite orientation.</li> <li>Since a line extends in <b>two opposite directions</b>, both sets correctly describe the <b>same line direction</b>.</li> <li>This demonstrates willingness to accept a new perspective: <b>direction is not unique; orientation may vary but the line is the same</b>.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |   |
| <b>Section C</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |   |
| 28.              | <p>A student believes that if two functions <math>f</math> and <math>g</math> are both one-one and onto, then <math>f \circ g = g \circ f</math>.</p> <p>Explain why this assumption may need reconsideration. Provide an example or reasoning to justify your explanation.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 3 |
|                  | <p><b>Point 1 : Identifies why the assumption is questionable (1 M)</b></p> <ul style="list-style-type: none"> <li>Even if both <math>f</math> and <math>g</math> are <b>bijective</b>, it does <b>not</b> imply<br/> <math display="block">f \circ g = g \circ f.</math></li> <li>The belief that “bijective <math>\Rightarrow</math> commutative composition” is <b>mathematically incorrect</b>.</li> <li>Shows openness to revising the initial assumption.</li> </ul> <p><b>Point 2 : Provides conceptual reasoning supporting the revision (1 M)</b></p> <ul style="list-style-type: none"> <li>Compositions depend on <b>order of application</b>; even when both functions are invertible, the output can differ depending on the order.</li> <li>Only special pairs of functions (e.g., identity, scalar multiples, certain symmetries) commute.</li> <li>Therefore, the student’s belief was based on an overgeneralisation.</li> </ul> <p><b>Point 3 : Provides a correct example demonstrating noncommutativity (1 M)</b></p> <p><b>Example</b><br/> Let</p> $f(x) = x + 1, g(x) = 2x.$ <p>Compute compositions:</p> $\begin{aligned} f(g(x)) &= f(2x) = 2x + 1, \\ g(f(x)) &= g(x + 1) = 2x + 2. \end{aligned}$ <p>Since</p> $2x + 1 \neq 2x + 2,$ <p>we have</p> $f \circ g \neq g \circ f.$ |   |

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|     | This contradicts the original belief and supports the need to revise it.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |   |
| 29. | <p>Prove that <math>2 \tan^{-1} x = \begin{cases} \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) &amp; \text{if } 0 \leq x \leq \infty \\ \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) &amp; \text{if } -\infty \leq x \leq 0 \end{cases}</math></p> <p><b>Point 1 : Begins with correct substitution and identity use (1 M)</b><br/> <math>\theta = \tan^{-1}(x) \Rightarrow \tan \theta = x</math>,</p> <p>and then correctly uses the identity</p> $\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ <p>or a geometric construction leading to</p> $\cos(2\theta) = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}.$ <p><b>Point 2 : Correctly expresses <math>\cos(2\theta)</math> in terms of <math>x</math> (1 M)</b></p> $\cos(2\theta) = \frac{1 - x^2}{1 + x^2}.$ <p>Then uses</p> $2 \tan^{-1}(x) = 2\theta,$ <p>so</p> $\cos(2 \tan^{-1}(x)) = \frac{1 - x^2}{1 + x^2}.$ <p><b>Point 3 : Applies range logic to determine correct principal values (1 M)</b></p> <ul style="list-style-type: none"> <li>For <math>x \geq 0</math>:</li> </ul> $\theta = \tan^{-1}(x) \in [0, \frac{\pi}{2}), 2\theta \in [0, \pi),$ <p>so</p> $2 \tan^{-1}(x) = \cos^{-1} \left( \frac{1 - x^2}{1 + x^2} \right).$ <ul style="list-style-type: none"> <li>For <math>x \leq 0</math>:</li> </ul> $\theta \in (-\frac{\pi}{2}, 0], 2\theta \in (-\pi, 0],$ | 3 |
| 30. | <p>For the differential equation</p> $(2xy + y^2)dx + x(x + y) dy = 0,$ <p>One student claims it can be solved by the method of variable separable form, while another student argues it is homogeneous.</p> <p>Analyze the structure of the equation and explain which method is appropriate.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 3 |

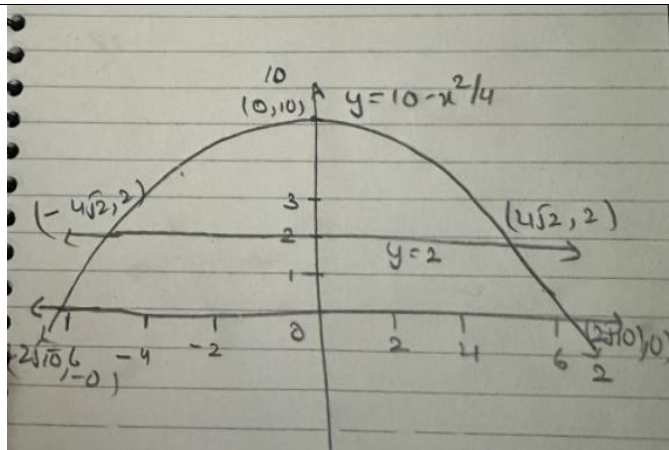
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |   |
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|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |   |
|     | <p><b>Point 1 : Shows correct structural analysis of terms (1 M)</b></p> <ul style="list-style-type: none"> <li>Each term <math>2xy</math>, <math>y^2</math>, <math>x^2</math>, and <math>xy</math> is of <b>degree 2</b> in variables <math>x</math> and <math>y</math>.</li> <li>Therefore, both expressions<br/> <math display="block">M(x, y) = 2xy + y^2, N(x, y) = x(x + y) = x^2 + xy</math> </li> </ul> <p>are <b>homogeneous functions of the same degree (degree 2)</b>.</p> <p><b>Point 2 : Explains why separation of variables does not apply (1 M)</b></p> <ul style="list-style-type: none"> <li>The equation cannot be written in the form <math>f(x) dx + g(y) dy = 0</math>.</li> <li>Variables appear <b>mixed</b> in every term, preventing separation.</li> <li>Hence, the first student's claim is incorrect <i>based on structure</i>.</li> </ul> <p><b>Point 3 : Concludes with correct method and conceptual justification (1 M):</b></p> <ul style="list-style-type: none"> <li>Since both <math>M</math> and <math>N</math> are homogeneous of the same degree, the <b>homogeneous differential equation method</b> is appropriate.</li> <li>The substitution <math>y = vx</math> (or <math>x = vy</math>) will reduce the equation to a separable form <b>after</b> homogenisation.</li> <li>This resolves the disagreement: the equation is homogeneous, not separable <i>as given</i>.</li> </ul> |   |
| 31. | <p>Two vectors are</p> $\vec{u} = (2, -1, 3), \vec{v} = (1, 4, -2).$ <p>Describe the steps used to decide whether the angle between them is acute, right, or obtuse, and determine the classification.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 3 |
|     | <p><b>Point 1 : States the correct procedural criterion (1 M)</b></p> $\vec{u} \cdot \vec{v} =  \vec{u}   \vec{v}  \cos \theta$ <p>and hence:</p> <ul style="list-style-type: none"> <li>If <math>\vec{u} \cdot \vec{v} &gt; 0</math>: angle is acute</li> <li>If <math>\vec{u} \cdot \vec{v} = 0</math>: angle is right</li> <li>If <math>\vec{u} \cdot \vec{v} &lt; 0</math>: angle is obtuse</li> </ul> <p><b>Point 2 : Performs the dot product calculation correctly (1 M)</b></p> $\begin{aligned} \vec{u} \cdot \vec{v} &= (2)(1) + (-1)(4) + (3)(-2) \\ &= 2 - 4 - 6 = -8. \end{aligned}$ <p><b>Point 3 : Interprets result using the process and gives correct classification (1 M)</b></p> <ul style="list-style-type: none"> <li>Since <math>\vec{u} \cdot \vec{v} = -8 &lt; 0</math>,<br/> <math>\Rightarrow \cos \theta &lt; 0</math><br/> <math>\Rightarrow</math> The angle between the vectors is obtuse.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |   |
| 32. | Find the angle between the lines whose direction cosine are given by the equations $3l + m + 5n = 0$ , and $6mn - 2nl + 5lm = 0$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | 3 |
|     | <b>Point 1 : Identifies the system as conditions on direction cosines (1 M)</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |   |

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|     | <ul style="list-style-type: none"> <li>The first equation:<br/> <math display="block">3l + m + 5n = 0</math> </li> </ul> <p>is of the form <math>al + bm + cn = 0</math>, indicating a <b>normal relation</b>.</p> <ul style="list-style-type: none"> <li>The second equation:<br/> <math display="block">6mn - 2nl + 5lm = 0</math> </li> </ul> <p>is the condition under which the direction cosines satisfy the <b>orthogonality relation</b>, implying <b>two lines are perpendicular</b>.</p> <p><b>Point 2 : Recognizes and states the perpendicularity principle (1 M)</b></p> <ul style="list-style-type: none"> <li>The condition<br/> <math display="block">6mn - 2nl + 5lm = 0</math> </li> </ul> <p>implies<br/> <math display="block">l_1l_2 + m_1m_2 + n_1n_2 = 0,</math> </p> <p>which means the <b>angle between the two lines is <math>90^\circ</math></b>.</p> <p><b>Point 3 : Concludes the required angle (1 M)</b><br/>       :</p> <ul style="list-style-type: none"> <li>Angle <math>\theta = 90^\circ</math> or <math>\theta = \frac{\pi}{2}</math>.</li> <li>Justifies via the dot product rule:<br/> <math display="block">\cos \theta = 0 \Rightarrow \theta = 90^\circ.</math> </li> </ul> |   |
| 33. | <p>Two railway tracks are represented by skew lines in a 3D design software. An engineer wants to build a safety tunnel so that its central axis lies along a line that is perpendicular to both tracks and meets the track design constraints. The lines are given by the equations:</p> $L_1: \frac{x}{1} = \frac{y-2}{2} = \frac{z}{-1}, \quad L_2: \frac{x-4}{2} = \frac{y}{1} = \frac{z-3}{2}.$ <p>Suggest a reasonable direction vector for the tunnel's central axis and explain why your chosen vector is suitable.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 3 |
|     | <p><b>Point 1 : Identifies direction vectors of the given skew lines (1 M)</b><br/>       From the symmetric forms:</p> <ul style="list-style-type: none"> <li>For <math>L_1</math>, direction vector:<br/> <math display="block">\vec{d}_1 = \langle 1, 2, -1 \rangle</math> </li> <li>For <math>L_2</math>, direction vector:<br/> <math display="block">\vec{d}_2 = \langle 2, 1, 2 \rangle</math> </li> </ul> <p><b>Point 2 : Generates an innovative, non routine idea using cross product (1 M)</b><br/>       A line perpendicular to both tracks must have a direction vector orthogonal to both <math>\vec{d}_1</math> and <math>\vec{d}_2</math>.<br/>       Thus compute or identify the cross product:</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |   |

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|                  | $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 2 & 1 & 2 \end{vmatrix} = \langle 5, -4, -3 \rangle.$ <p>Any nonzero scalar multiple (e.g., <math>\langle 5, -4, -3 \rangle</math>, <math>\langle -5, 4, 3 \rangle</math>, <math>\langle 10, -8, -6 \rangle</math>) earns full credit.</p> <p><b>Point 3 — Explains why the proposed vector is suitable in the engineering context (1 M)</b><br/> Full credit if the student explains:</p> <ul style="list-style-type: none"> <li>The cross-product vector is <b>orthogonal to both track directions</b>,</li> <li>Therefore, a tunnel built along this vector satisfies the requirement of being perpendicular to both skew railway tracks,</li> <li>This provides a <b>structurally efficient and collision avoidant</b> direction for the central tunnel axis.</li> </ul> |   |
| <b>Section D</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |   |
| 34.              | <p>A system of equations is given by</p> $\begin{aligned} 2x + 3y &= 7, \\ 4x + ky &= 14. \end{aligned}$ <p>(a) Use determinants to determine the value(s) of <math>k</math> for which the system has:</p> <ul style="list-style-type: none"> <li>a unique solution</li> <li>infinitely many solutions</li> <li>no solution</li> </ul> <p>(b) Explain how the determinant of the coefficient matrix determines these outcomes.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                               | 4 |
|                  | <p><b>Point 1 : Computes the determinant of the coefficient matrix (1 M)</b><br/> Coefficient matrix:</p> $A = \begin{pmatrix} 2 & 3 \\ 4 & k \end{pmatrix}$ <p>Determinant:</p> $\Delta = \det(A) = 2k - 12 = 2(k - 6).$ <p><b>Point 2 : Uses determinant conditions to classify the system (1½ Ms)</b><br/> <b>Unique Solution (½ M)</b><br/> Occurs when <math>\Delta \neq 0</math>.</p> $2(k - 6) \neq 0 \Rightarrow k \neq 6.$ <p><b>Infinitely Many Solutions (½ M)</b><br/> Occurs when <math>\Delta = 0</math> and the ratios match:</p> $\frac{2}{4} = \frac{3}{k} = \frac{7}{14}.$ $\frac{2}{4} = \frac{1}{2}, \frac{7}{14} = \frac{1}{2} \Rightarrow \frac{3}{k} = \frac{1}{2} \Rightarrow k = 6.$ <p>Thus infinitely many solutions <b>only when</b> <math>k = 6</math>.<br/> <b>No Solution (½ M)</b></p>                                                           |   |

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|     | <p>Occurs when <math>\Delta = 0</math> but ratios of coefficients fail:<br/> If <math>k = 6</math> and ratio of constants does <i>not</i> match, we would have no solution, but here</p> $\frac{2}{4} = \frac{3}{6} = \frac{7}{14},$ <p>so for this system there is <b>no value of <math>k</math></b> giving "no solution".</p> <p><b>Point 3 : Explanation linking determinant to solution behaviour (1 M)</b><br/> Award full credit when the student explains:</p> <ul style="list-style-type: none"> <li>The determinant of the coefficient matrix determines whether the rows are linearly independent.</li> <li>If <math>\Delta \neq 0</math>: rows independent <math>\rightarrow</math> lines intersect at one point <math>\rightarrow</math> <b>unique solution</b>.</li> <li>If <math>\Delta = 0</math>: rows dependent <math>\rightarrow</math> lines parallel or coincident <math>\rightarrow</math> <b>infinite or no solution</b> depending on constant ratios.</li> <li>Thus determinant acts as the <b>key diagnostic tool</b> linking algebraic structure to system behaviour.</li> </ul> <p><b>Point 4 : Coherent integration of concepts (½ M)</b><br/> Award this if the student <i>connects</i> determinants + linear independence + geometry + consistency, showing genuine <b>combination of ideas</b>, not isolated facts.</p> |   |
| 35. | <p>(a) State the relationship between continuity and differentiability of a function at a point.</p> <p>(b) Using first principles of derivatives, find the derivative of the function<br/> <math display="block">f(x) = \sqrt{x}</math> and specify the condition under which the derivative exists.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 4 |
|     | <p><b>Point 1 : States the correct conceptual relationship (1 M)</b><br/> Award <b>1 M</b> for stating:</p> <ul style="list-style-type: none"> <li>If a function is <b>differentiable</b> at a point, then it is <b>always continuous</b> at that point.</li> <li>The <b>converse is not necessarily true</b>: continuity does not guarantee differentiability.</li> </ul> <p><b>Point 2 : Writes the first principles formula correctly (½ M)</b><br/> Student writes:</p> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$ <p><b>Point 3 : Applies it correctly to <math>\sqrt{x}</math> (1½ Ms)</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |   |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |   |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---|
|     | <p>Step-by-step first principles derivation:</p> $f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ <p>Multiply numerator and denominator by the conjugate:</p> $= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$ $= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$ $= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$ <p>Taking the limit:</p> $f'(x) = \frac{1}{2\sqrt{x}}$ <p><b>Point 4 : States the condition for existence of the derivative (1 M)</b><br/> Student states:</p> <ul style="list-style-type: none"> <li>The derivative</li> </ul> $f'(x) = \frac{1}{2\sqrt{x}}$ <p>exists <b>only for</b> <math>x &gt; 0</math>.</p> <ul style="list-style-type: none"> <li>At <math>x = 0</math>:<br/> Left hand derivative does not exist (domain restriction),<br/> Right hand derivative tends to <math>+\infty</math>.</li> <li>Hence, <b>derivative exists for all</b> <math>x &gt; 0</math>.</li> </ul> |   |
| 36. | <p>A cross- sectional shape of a tunnel is modelled by the curve</p> $y = 10 - \frac{x^2}{4}.$ <p>Engineers plan to raise the tunnel floor by 2 m, shifting the ground line from <math>y = 0</math> to <math>y = 2</math>.</p> <p>Find the area of the tunnel cross- section after this elevation.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 4 |



**Point 1 : Sets up the new region and finds intersection points (1 M)**

Solve intersection with new floor line  $y = 2$ :

$$\begin{aligned} 10 - \frac{x^2}{4} &= 2 \\ \frac{x^2}{4} &= 8 \\ x^2 &= 32, x = \pm 4\sqrt{2}. \end{aligned}$$

Thus the tunnel Cross- section extends from  $-4\sqrt{2}$  to  $4\sqrt{2}$ .

**Point 2 — Formulates the correct definite integral for area (1 M)**

Area between curve and new floor line:

$$\begin{aligned} A &= \int_{-4\sqrt{2}}^{4\sqrt{2}} \left(10 - \frac{x^2}{4} - 2\right) dx \\ &= \int_{-4\sqrt{2}}^{4\sqrt{2}} \left(8 - \frac{x^2}{4}\right) dx. \end{aligned}$$

**Point 3 — Performs integration step-by-step (1 M)**

Integrate term by term:

$$\int \left(8 - \frac{x^2}{4}\right) dx = 8x - \frac{x^3}{12}.$$

Evaluate from  $-4\sqrt{2}$  to  $4\sqrt{2}$ :

Because integrand is even:

$$A = 2\left[8x - \frac{x^3}{12}\right]_0^{4\sqrt{2}}.$$

At  $x = 4\sqrt{2}$ :

$$8(4\sqrt{2}) = 32\sqrt{2}, (4\sqrt{2})^3 = 128\sqrt{2}.$$

$$\frac{x^3}{12} = \frac{128\sqrt{2}}{12} = \frac{32\sqrt{2}}{3}.$$

Thus:

$$A = 2(32\sqrt{2} - \frac{32\sqrt{2}}{3}) = 2(\frac{96\sqrt{2} - 32\sqrt{2}}{3}) = 2(\frac{64\sqrt{2}}{3}) = \frac{128\sqrt{2}}{3}.$$

**Point 4 — Final area statement with correct justification (1 M)**

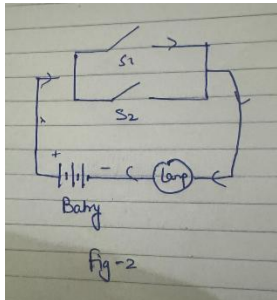
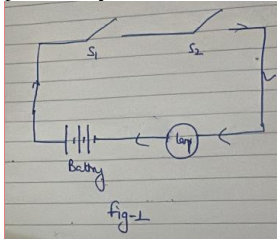
Area of raised tunnel Cross- section:

|                                               |
|-----------------------------------------------|
| $\frac{128\sqrt{2}}{3} \text{ square metres}$ |
|-----------------------------------------------|

Explanation: The area is the region between the curve  $y = 10 - x^2/4$  and the elevated floor line  $y = 2$ , found by definite integration over the intersection interval.

37.

If two switches S1 and S2 have 90% and 80% chance of working, respectively, find the probability that each of the following circuit will work.



4

Commented [PS1]: Reconstruct the images

Consider the following event:

A = Switch  $S_1$  works

B = Switch  $S_2$  works

$$P(A) = \frac{90}{100} = \frac{9}{10}$$

$$P(B) = \frac{80}{100} = \frac{8}{10}$$

} — 1M

(i) The circuit will work if the current flows in the circuit. This is possible only when both the switches work together. Therefore

$$\begin{aligned} \text{Required probability} &= P(A \cap B) \\ \text{Required Probability} &= P(A)P(B) \quad [A \text{ and } B \text{ are independent event}] \\ &= \frac{9}{10} \times \frac{8}{10} \\ &= \frac{72}{100} = \frac{18}{25} \end{aligned}$$

(ii) The circuit will work if the current flows in the circuit. This is possible only when at least one of two switches  $S_1$  or  $S_2$  works. Therefore,

$$\begin{aligned} \text{Required Probability} &= P(A \cup B) \\ \text{Required Probability} &= 1 - P(\bar{A})P(\bar{B}) \quad [A \text{ and } B \text{ are independent event}] \\ &= 1 - \left(1 - \frac{9}{10}\right) \left(1 - \frac{8}{10}\right) \\ &= 1 - \frac{1}{10} \times \frac{2}{10} = \frac{49}{50} \end{aligned}$$

38. A brick manufacture has two depots A and B with the stocks of 30,000 and 20,000 bricks, respectively. The manufacturer receives orders from three builder:
- P: 15,000 bricks
  - Q: 20,000 bricks
  - R: 15000 bricks respectively.

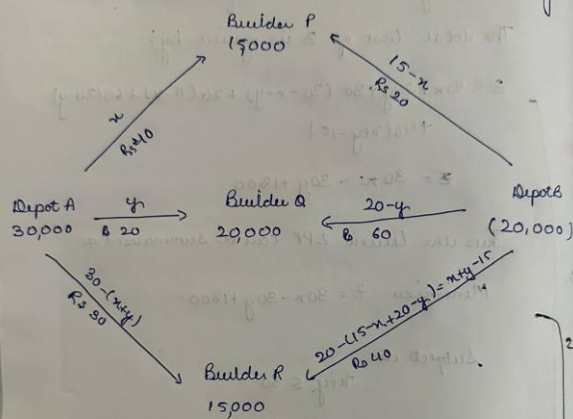
The cost (in R) transporting 1,000 bricks to the builder from each depot to each builder is given by:

| To/ From | P  | Q  | R  |
|----------|----|----|----|
| A        | 40 | 20 | 30 |
| B        | 20 | 60 | 40 |

How should the manufacturer allocate bricks from depots A and B to builders P, Q, and R so that the total transportation cost is minimized?

4

The given information can be represented diagrammatically



Let the depot A transport  $x$  thousand bricks to builder P  
 &  $y$  to builder Q. Since the depot A has 30,000 bricks. Therefore  
 the remaining bricks i.e.  $30-(x+y)$  thousand bricks will  
 be transported to the builder R

Therefore  $x \geq 0, y \geq 0$  &  $30-(x+y) \geq 0 \Rightarrow x+y \leq 30$

Now depot B has  $20-(15-x+20-y) = x+y-15$  thousand  
 bricks which can be transported to builder R

$$x+y-15 \geq 0 \Rightarrow x+y \geq 15$$

According to the given information

The total cost of  $z$  is given by:

$$z = 40x + 20y + 30(30 - x - y) + 20(15 - x) + 60(20 - y) + 40(x + y - 15)$$

$$z = 30x - 30y + 1800$$

Thus the given LPP can be summarised as

$$\text{Minimise } z = 30x - 30y + 1800$$

Subject to

$$x + y \leq 30$$

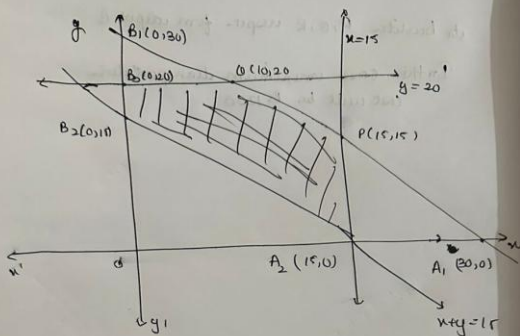
$$x \leq 15$$

$$y \leq 20$$

$$x + y \geq 15$$

$$x \geq 0, y \geq 0$$

The feasible region of LPP is given shaded below



The value of objective function at the corner point of feasible region is given as

|             |                                                |
|-------------|------------------------------------------------|
| $A_2(15,0)$ | $Z = 30 \times 15 - 30 \times 0 + 1500 = 2250$ |
| $P(15,15)$  | $= 1800$                                       |
| $B(10,20)$  | $= 1500$                                       |
| $B_3(0,20)$ | $= 1200$                                       |
| $B_2(0,15)$ | $= 1350$                                       |

$Z$  is minimum when  $x=0$   $y=0$  (the minimum value of  $Z = 1200$ )

The manufacturer should supply 0,20,10 thousand bricks from P, O, R from depot A 15,0 25 thousand bricks