UNIT 13

PLAYING WITH NUMBERS

(A) Main Concepts and Results

- Numbers can be written in general form. For example, a two digit number $ab$ is written as $ab = 10a + b$; a three digit number $abc$ is written as $abc = 100a + 10b + c$.
- The general form of numbers are helpful in solving various problems related to numbers.
- Rationale for the divisibility of numbers by 11, 10, 5, 2, 9 or 3 can be explained by writing the numbers in general form.
- Many number puzzles involving different letters for different digits are solved using rules of number operations.

(B) Solved Examples

In examples 1 to 4, out of four options only one is correct. Write the correct answer.

**Example 1** : Generalised form of a three-digit number $xyz$ is

(a) $x + y + z$  
(b) $100x + 10y + z$

(c) $100z + 10y + x$  
(d) $100y + 10x + z$

**Solution** : The correct answer is (b).

**Example 2** : The usual form of $100a + b + 10c$ is

(a) $abc$  
(b) $cab$  
(c) $bac$  
(d) $acb$

**Solution** : The correct answer is (d).

**Example 3** : If $5 \times A = CA$ then the values of A and C are
(a)  A = 5,  C = 1
(b)  A = 4,  C = 2
(c)  A = 5,  C = 2
(d)  A = 2,  C = 5

Solution  : The correct answer is (c).

Example 4 : If 5 A + 25 is equal to B 2, then the value of A + B is
(a)  15 (b)  10 (c)  8 (d)  7

Solution  : The correct answer is (a).

In examples 5 to 7, fill in the blanks to make the statements true.

Example 5 : The number $ab - ba$ where $a$ and $b$ are digits and $a > b$ is divisible by ________.

Solution  : 9.

Example 6 : When written in usual form $100a + 10c + 9$ is equal to ________.

Solution  : $ac 9$

Example 7 : If $AB \times B = 9B$, then $A = ________$, $B = ________$.

Solution  : 9, 1

In examples 8 to 10, state whether the statements are true (T) or false (F).

Example 8 : If $abc, cab, bca$ are three digit numbers formed by the digits $a, b$, and $c$ then the sum of these numbers is always divisible by 37.

Solution  : True.

Example 9 : Let $ab$ be a two-digit number, then $ab + ba$ is divisible by 9.

Solution  : False.

Example 10 : If a number is divisible by 2 and 4, then it will be divisible by 8.

Solution  : False.

Example 11 : A three-digit number $42x$ is divisible by 9. Find the value of $x$.

Solution  : Since $42x$ is divisible by 9, the sum of its digits, i.e. $4 + 2 + x$ must be divisible by 9.
i.e. \(6 + x\) is divisible by 9
i.e. \(6 + x = 9\) or \(18\), _____.
Since \(x\) is a digit, therefore \(6 + x = 9\) or \(x = 3\).

**Example 12**: Find the value of \(A\) and \(B\) if

\[
\begin{array}{c}
41 \ A \\
+ \ B \\
\hline
5 \ 1 \ 2
\end{array}
\]

**Solution**: From ones column \(A + 4\) gives a number whose ones digit is 2. So, \(A = 8\). The value of \(B\) can be obtained by solving \(2 + B\) is a number whose ones digit is 1. So, \(B = 9\).

\[
\begin{array}{c}
418 \\
+ \ 94 \\
\hline
512
\end{array}
\]

**Example 13**: Suppose that the division \(x \div 5\) leaves a remainder 4 and the division \(x \div 2\) leaves a remainder 1. Find the ones digit of \(x\).

**Solution**: Since \(x \div 5\) leaves a remainder 4, so ones digit of \(x\) can be 4 or 9. Also, since \(x \div 2\) leaves a remainder 1, so ones digit must be 9 only.

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**Application on Problem Solving Strategy**

**Example 14**: If \(756x\) is divisible by 11, where \(x\) is a digit find the value of \(x\).

**Understand and Explore the problem**

- What is given in the question?
  A four digit number \(756x\) is divisible by 11.
- Which property is required to solve the problem?
  Divisibility of a number by 11.

**Plan a Strategy**

- Find the sum of the digits of given number \(756x\) at odd places.
- Find the sum of the digits of \(756x\) at even places.
- Find the difference of step 1 and step 2.
In each of the questions 1 to 17, out of the four options, only one is correct. Write the correct answer.

1. Generalised form of a four-digit number $abdc$ is
   
   (a) $1000a + 100b + 10c + d$
   (b) $1000a + 100c + 10b + d$
   (c) $1000a + 100b + 10d + c$
   (d) $a \times b \times c \times d$
2. Generalised form of a two-digit number $xy$ is
   (a) $x + y$   (b) $10x + y$   (c) $10x - y$   (d) $10y + x$

3. The usual form of $1000a + 10b + c$ is
   (a) $abc$   (b) $abco$   (c) $aobc$   (d) $aboc$

4. Let $abc$ be a three-digit number. Then $abc - cba$ is not divisible by
   (a) 9   (b) 11   (c) 18   (d) 33

5. The sum of all the numbers formed by the digits $x$, $y$ and $z$ of the number $xyz$ is divisible by
   (a) 11   (b) 33   (c) 37   (d) 74

6. A four-digit number $aabb$ is divisible by 55. Then possible value(s) of $b$ is/are
   (a) 0 and 2   (b) 2 and 5   (c) 0 and 5   (d) 7

7. Let $abc$ be a three digit number. Then $abc + bca + cab$ is not divisible by
   (a) $a + b + c$   (b) 3   (c) 37   (d) 9

8. A four-digit number $4ab5$ is divisible by 55. Then the value of $b - a$ is
   (a) 0   (b) 1   (c) 4   (d) 5

9. If $abc$ is a three digit number, then the number $abc - a - b - c$ is divisible by
   (a) 9   (b) 90   (c) 10   (d) 11

10. A six-digit number is formed by repeating a three-digit number. For example 256256, 678678, etc. Any number of this form is divisible by
    (a) 7 only   (b) 11 only   (c) 13 only   (d) 1001

11. If the sum of digits of a number is divisible by three, then the number is always divisible by
    (a) 2   (b) 3   (c) 6   (d) 9

12. If $x + y + z = 6$ and $z$ is an odd digit, then the three-digit number $xyz$ is
    (a) an odd multiple of 3   (b) odd multiple of 6
    (c) even multiple of 3   (d) even multiple of 9
13. If $5A + B = 3 = 65$, then the value of $A$ and $B$ is
   (a) $A = 2$, $B = 3$  
   (b) $A = 3$, $B = 2$  
   (c) $A = 2$, $B = 1$  
   (d) $A = 1$, $B = 2$

14. If $A + 8B = 150$, then the value of $A + B$ is
   (a) 13  
   (b) 12  
   (c) 17  
   (d) 15

15. If $5A \times A = 399$, then the value of $A$ is
   (a) 3  
   (b) 6  
   (c) 7  
   (d) 9

16. If $6A \times B = A 8B$, then the value of $A - B$ is
   (a) 2  
   (b) 2  
   (c) -3  
   (d) 3

17. Which of the following numbers is divisible by 99
   (a) 913462  
   (b) 114345  
   (c) 135792  
   (d) 3572406

In questions 18 to 33, fill in the blanks to make the statements true.

18. 3134673 is divisible by 3 and ______.

19. $20x3$ is a multiple of 3 if the digit $x$ is ______ or ______ or ______.

20. $3x5$ is divisible by 9 if the digit $x$ is ______.

21. The sum of a two-digit number and the number obtained by reversing the digits is always divisible by ______.

22. The difference of a two-digit number and the number obtained by reversing its digits is always divisible by ______.

23. The difference of three-digit number and the number obtained by putting the digits in reverse order is always divisible by 9 and ______.

24. If \[
\begin{array}{c}
2 & B \\
A & 8
\end{array}
\] then $A = _____$ and $B = _____$.

25. If \[
\begin{array}{c}
A & B \\
9 & 6
\end{array}
\] then $A = _____$ and $B = _____$.

26. If \[
\begin{array}{c}
B & 1 \\
4 & 9B
\end{array}
\] then $B = _____$. 

In questions 18 to 33, fill in the blanks to make the statements true.
27. 1 \times 35 \text{ is divisible by } 9 \text{ if } x = \underline{\phantom{0}}.

28. A four-digit number \( abcd \) is divisible by 11, if \( d + b = \underline{\phantom{0}} \) or \( \underline{\phantom{0}} \).

29. A number is divisible by 11 if the differences between the sum of digits at its odd places and that of digits at the even places is either 0 or divisible by \( \underline{\phantom{0}} \).

30. If a 3-digit number \( abc \) is divisible by 11, then \( \underline{\phantom{0}} \) is either 0 or multiple of 11.

31. If \( A \times 3 = 1A \), then \( A = \underline{\phantom{0}} \).

32. If \( B \times B = AB \), then either \( A = 2 \), \( B = 5 \) or \( A = \underline{\phantom{0}} \), \( B = \underline{\phantom{0}} \).

33. If the digit 1 is placed after a 2-digit number whose tens is \( t \) and ones digit is \( u \), the new number is \( \underline{\phantom{0}} \).

State whether the statements given in questions 34 to 44 are true (T) or false (F):

34. A two-digit number \( ab \) is always divisible by 2 if \( b \) is an even number.

35. A three-digit number \( abc \) is divisible by 5 if \( c \) is an even number.

36. A four-digit number \( abcd \) is divisible by 4 if \( ab \) is divisible by 4.

37. A three-digit number \( abc \) is divisible by 6 if \( c \) is an even number and \( a + b + c \) is a multiple of 3.

38. Number of the form \( 3N + 2 \) will leave remainder 2 when divided by 3.

39. Number \( 7N + 1 \) will leave remainder 1 when divided by 7.

40. If a number \( a \) is divisible by \( b \), then it must be divisible by each factor of \( b \).

41. If \( AB \times 4 = 192 \), then \( A + B = 7 \).

42. If \( AB + 7C = 102 \), where \( B \neq 0, \ C \neq 0 \), then \( A + B + C = 14 \).

43. If \( 213x27 \) is divisible by 9, then the value of \( x \) is 0.

44. If \( N \div 5 \) leaves remainder 3 and \( N \div 2 \) leaves remainder 0, then \( N \div 10 \) leaves remainder 4.

Solve the following:

45. Find the least value that must be given to number \( a \) so that the number \( 91876a2 \) is divisible by 8.
46. If \( \frac{1}{P} \times P \) where \( Q - P = 3 \), then find the values of \( P \) and \( Q \).

47. If \( 1AB + CCA = 697 \) and there is no carry-over in addition, find the value of \( A + B + C \).

48. A five-digit number \( AABAA \) is divisible by 33. Write all the numbers of this form.

49. Find the value of the letters in each of the following questions.

\[
\begin{array}{ccc}
\text{A} & \text{A} \\
+ & \text{A} & \text{A} \\
\hline
\text{X} & \text{A} & \text{Z}
\end{array}
\]

50. \[ \begin{array}{ccc}
8 & 5 \\
+ & 4 & A \\
\hline
B & C & 3
\end{array} \]

51. \[ \begin{array}{ccc}
B & 6 \\
+ & 8 & A \\
\hline
C & A & 2
\end{array} \]

52. \[ \begin{array}{ccc}
1 & B & A \\
+ & A & B & A \\
\hline
& & 8 & A & 2
\end{array} \]

53. \[ \begin{array}{ccc}
C & B & A \\
+ & C & B & A \\
\hline
1 & A & 3 & 0
\end{array} \]

54. \[ \begin{array}{ccc}
B & A & A \\
+ & B & A & A \\
\hline
3 & A & 8
\end{array} \]

55. \[ \begin{array}{ccc}
A & 0 & 1 & B \\
+ & 1 & 0 & A & B \\
\hline
& & B & 1 & 0 & 8
\end{array} \]

56. \[ \begin{array}{ccc}
A & B \\
\times & 6 \\
\hline
C & 6 & 8
\end{array} \]

57. \[ \begin{array}{ccc}
A & B \\
\times & A & B \\
\hline
& & 6 & A & B
\end{array} \]

58. \[ \begin{array}{ccc}
A & A \\
\times & A \\
\hline
& & C & A & B
\end{array} \]

and \( B - A = 1 \)

59. \[ \begin{array}{ccc}
A & B \\
- & B & 7 \\
\hline
& & 4 & 5
\end{array} \]

60. \[ \begin{array}{ccc}
8 & A & B & C \\
- & A & B & C & 5 \\
\hline
& & D & 4 & 8 & 8
\end{array} \]

61. If \( 2A7 \div A = 33 \), then find the value of \( A \).

62. \( 212 \times 5 \) is a multiple of 3 and 11. Find the value of \( x \).

63. Find the value of \( k \) where \( 31k2 \) is divisible by 6.

64. \( 1y3y6 \) is divisible by 11. Find the value of \( y \).

65. \( 756 \times x \) is a multiple of 11, find the value of \( x \).

66. A three-digit number \( 2a3 \) is added to the number 326 to give a three-digit number \( 5b9 \) which is divisible by 9. Find the value of \( b - a \).
67. Let \( E = 3, B = 7 \) and \( A = 4 \). Find the other digits in the sum
\[
\begin{array}{c}
B & A & S & E \\
+ & B & A & L & L \\
\hline
G & A & M & E & S
\end{array}
\]
68. Let \( D = 3, L = 7 \) and \( A = 8 \). Find the other digits in the sum
\[
\begin{array}{c}
M & A & D \\
+ & A & S \\
+ & A \\
\hline
B & U & L & L
\end{array}
\]
69. If from a two-digit number, we subtract the number formed by reversing its digits then the result so obtained is a perfect cube. How many such numbers are possible? Write all of them.

70. Work out the following multiplication.
\[
12345679 \\
\times 9
\]
Use the result to answer the following questions.
(a) What will be \( 12345679 \times 45 \)?
(b) What will be \( 12345679 \times 63 \)?
(c) By what number should \( 12345679 \) be multiplied to get 888888888?
(d) By what number should \( 12345679 \) be multiplied to get 999999999?

71. Find the value of the letters in each of the following:
(i) \( PQ \times 6 \)
(ii) \( 2LM + LMI \)

72. If 148101B095 is divisible by 33, find the value of \( B \).

73. If 123123A4 is divisible by 11, find the value of \( A \).

74. If 56x32y is divisible by 18, find the least value of \( y \).
1. Polygonal Numbers

Study the patterns given below and extend it. We already know about square numbers.

\[
\begin{array}{cccc}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{array}
\]

| 1 | 4 | 9 |

Draw two more.

Here for the first square number, use \(1^2\); for the second square number, use \(2^2\). To find the third square number use \(3^2\) and so on. Write the \(n\)th square number.

Now let’s move to triangular numbers.

\[
\begin{array}{cccc}
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{array}
\]

| 1 | 3 | 6 | 10 |

Find the next triangular number.

To find the \(n\)th triangular number we use the formula \(\frac{n \times (n+1)}{2}\)
Are you familiar with pentagonal numbers?

First three are given to you. Write the next one

\[
\begin{array}{c}
* & * & * & * & * \\
1 & * & * & * & * \\
& * & * & * & * & * \\
5 & * & * & * & * \\
& & & & & \\
& 12
\end{array}
\]

Draw the dot patterns for the next pentagonal number. Count the number of dots inside the entire shape and write the number under the shape.

2. Put tick mark in the appropriate boxes if the given numbers are divisible by any of 2, 3, 4, 5, 6, 8, 10, 11 numbers.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Number</th>
<th>Divisible by</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>40185</td>
<td>2 3 4 5 6</td>
</tr>
<tr>
<td>2.</td>
<td>92286</td>
<td>2 3 4 5 6</td>
</tr>
<tr>
<td>3.</td>
<td>56390</td>
<td>2 3 4 5 6</td>
</tr>
<tr>
<td>4.</td>
<td>419562</td>
<td>2 3 4 5 6</td>
</tr>
<tr>
<td>5.</td>
<td>10593248</td>
<td>2 3 4 5 6</td>
</tr>
</tbody>
</table>
3. Cross Number Puzzle

Fill in the blank spaces in the cross number puzzle using following clues.

Down

(a) 59 ______ 63 ÷ 33
(b) 81 ______ 42 ÷ 6
(c) 7 ______ 6988 ÷ 11
(d) 37604 _____ 5 ÷ 15
(e) 56 _____ ÷ 10

Across

(f) 90 _____ 815 ÷ 15
(g) 3514 _____ ÷ 12
(h) 4 _____ 07 ÷ 7

   (i) 8 _____ 558 ÷ 6
   (j) 6 _____ 5 ÷ 55
Rough Work